Neural Networks

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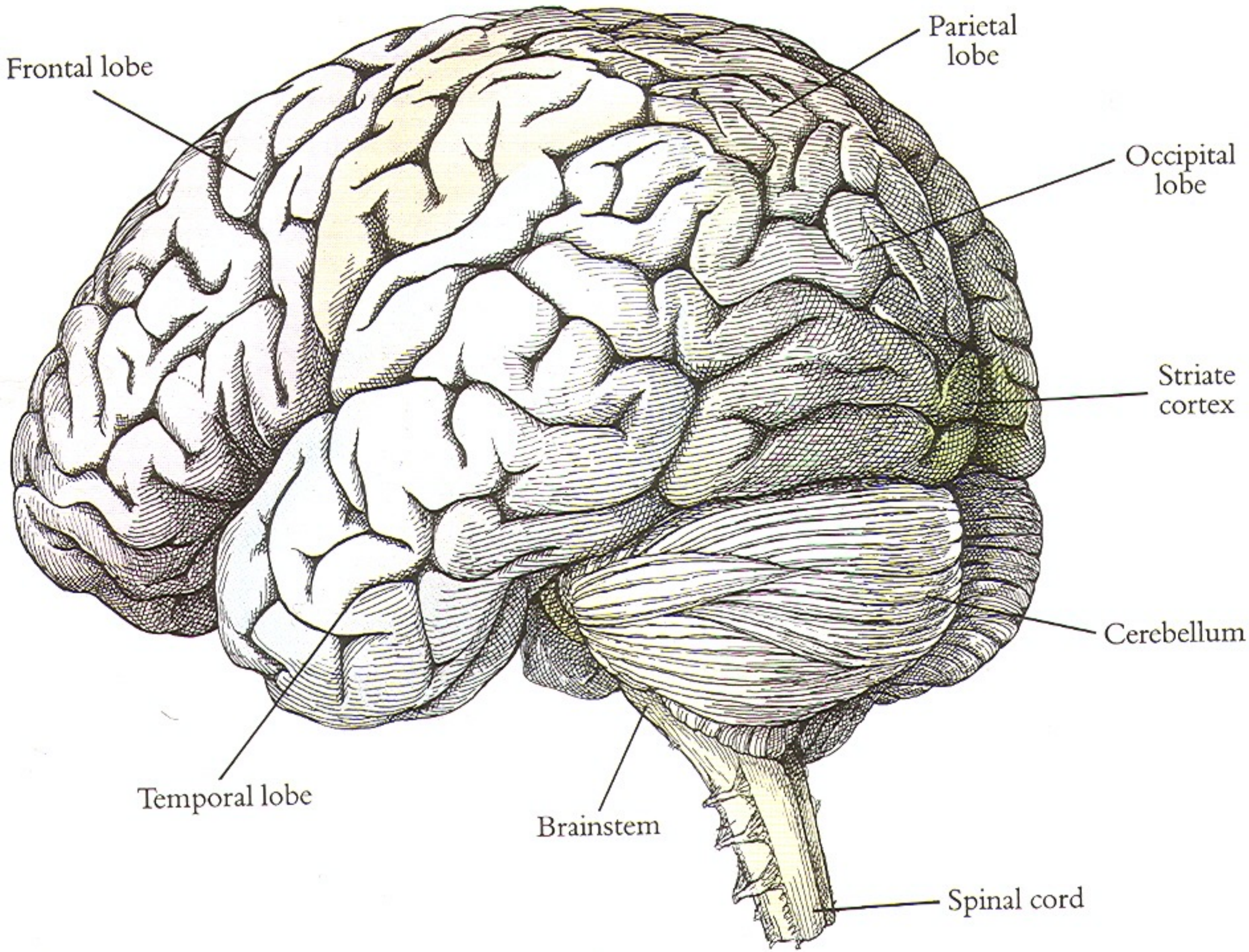
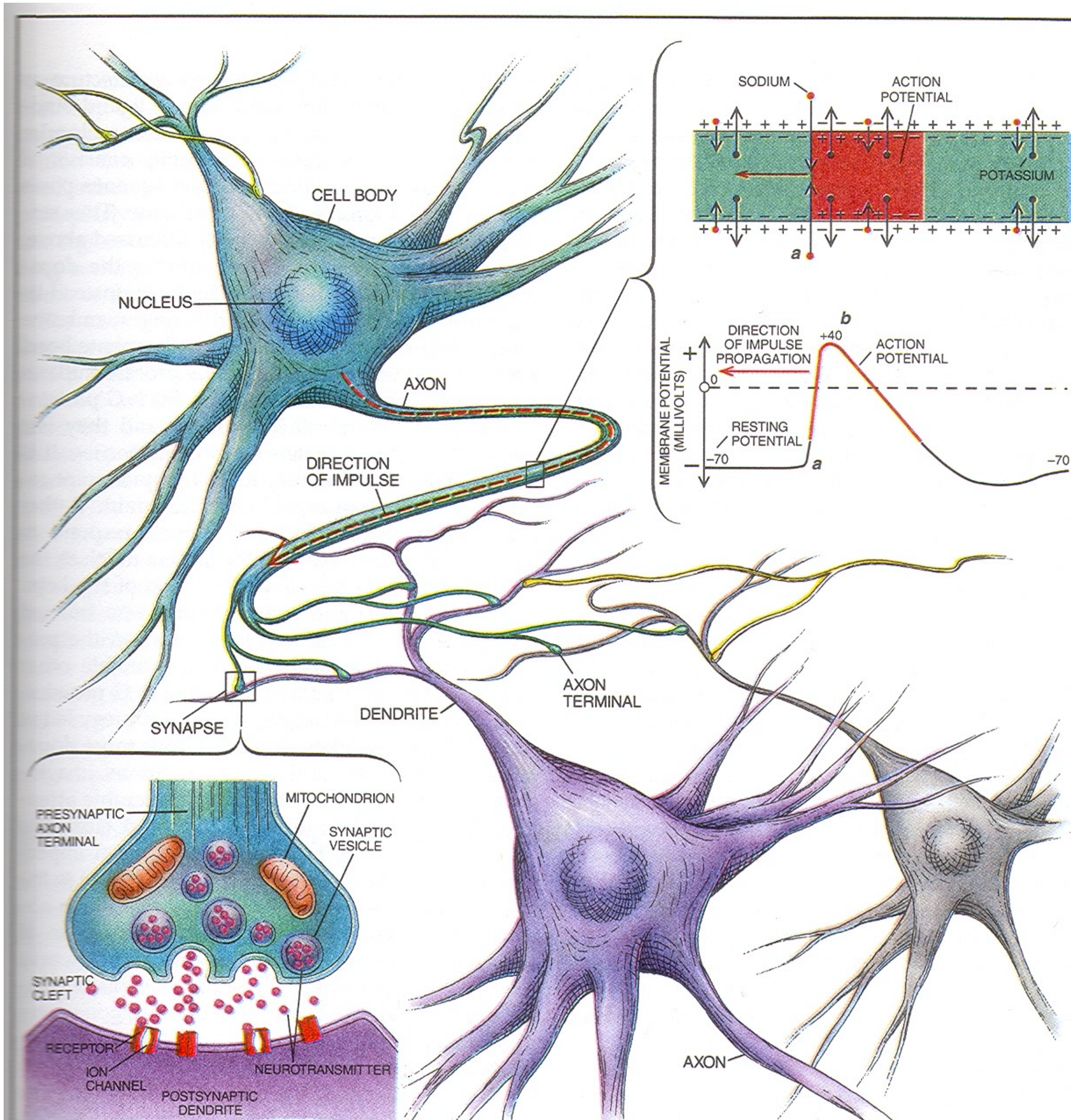
CS 189


Frontal lobe
Parietal lobe
Occipital lobe
Striate cortex
Cerebellum
Temporal lobe
Brainstem
Spinal cord

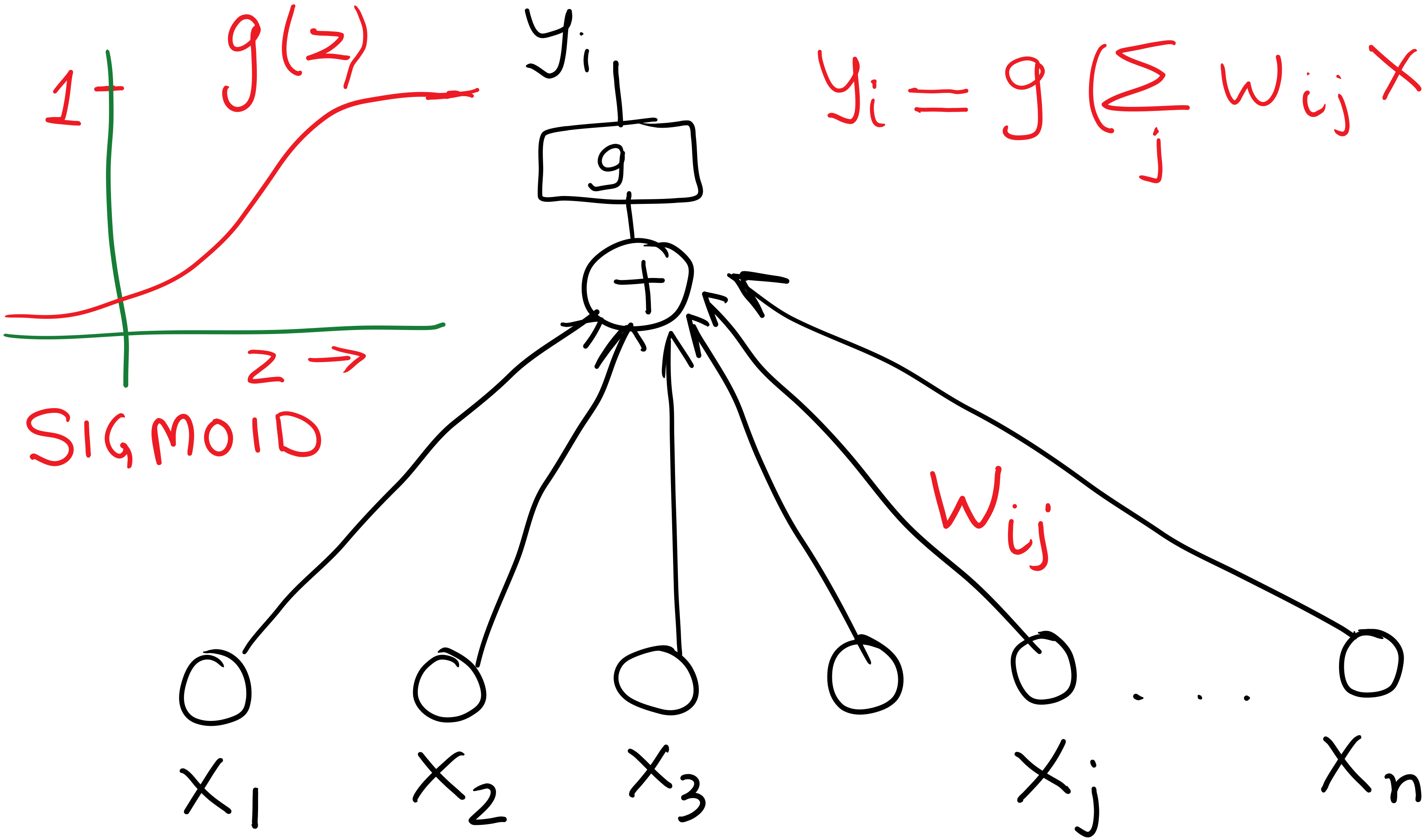
Key facts

- The human brain has ~ 100 billion neurons (cells)
- Most neurons input signals via the dendrites and output signals via the axon
- At the tip of an axon’s branches, there are axon terminals, where the neuron can transmit a signal across the synapse to another cell
- Typically the signal flow along an axon is in the form of voltage spikes, with more spikes per second indicating a stronger output



How Neurons Communicate

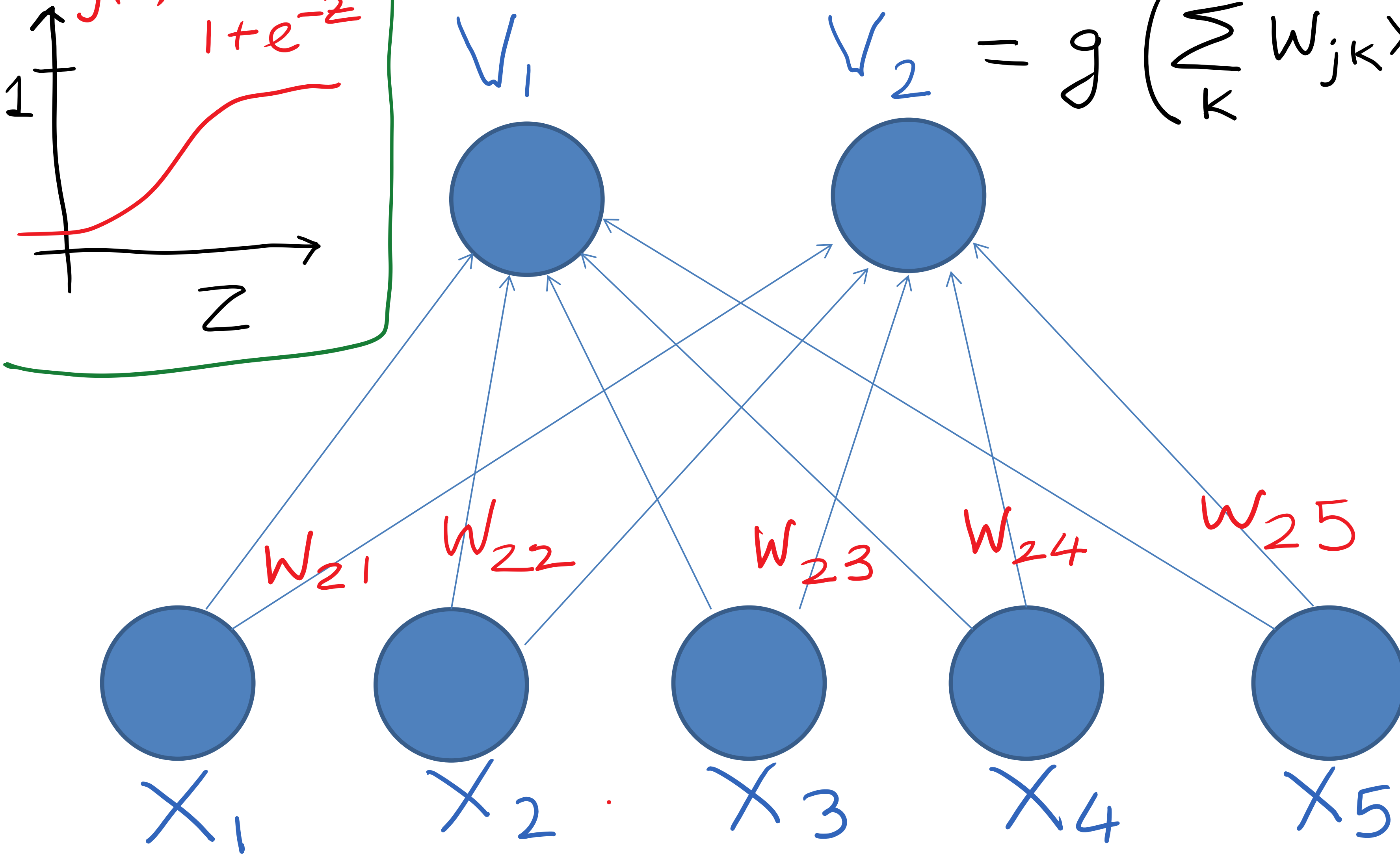
Mathematical Abstraction



$$y_i = g\left(\sum_j w_{ij} x_j\right)$$

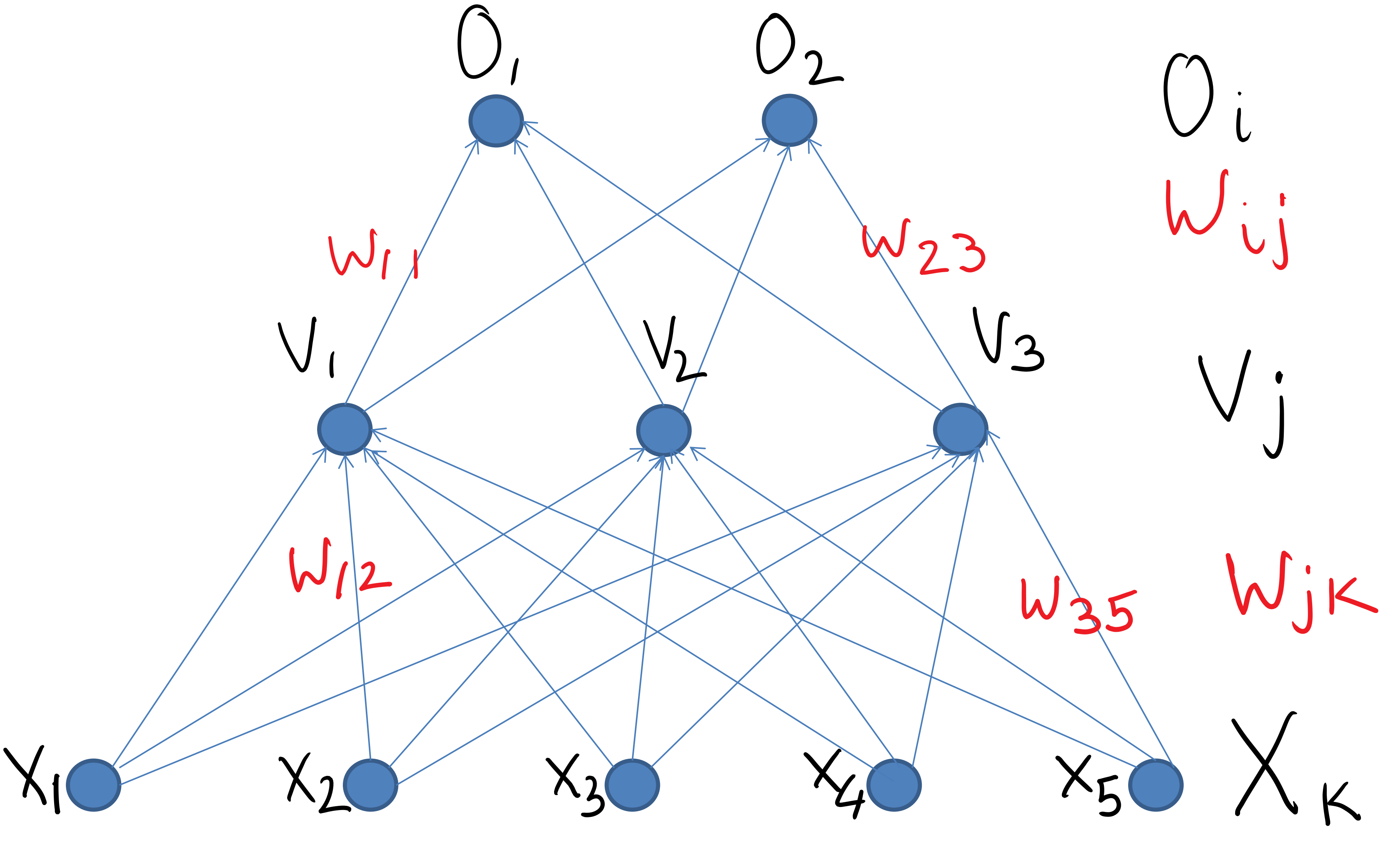
Single layer neural network

$$g(z) = \frac{1}{1+e^{-z}}$$

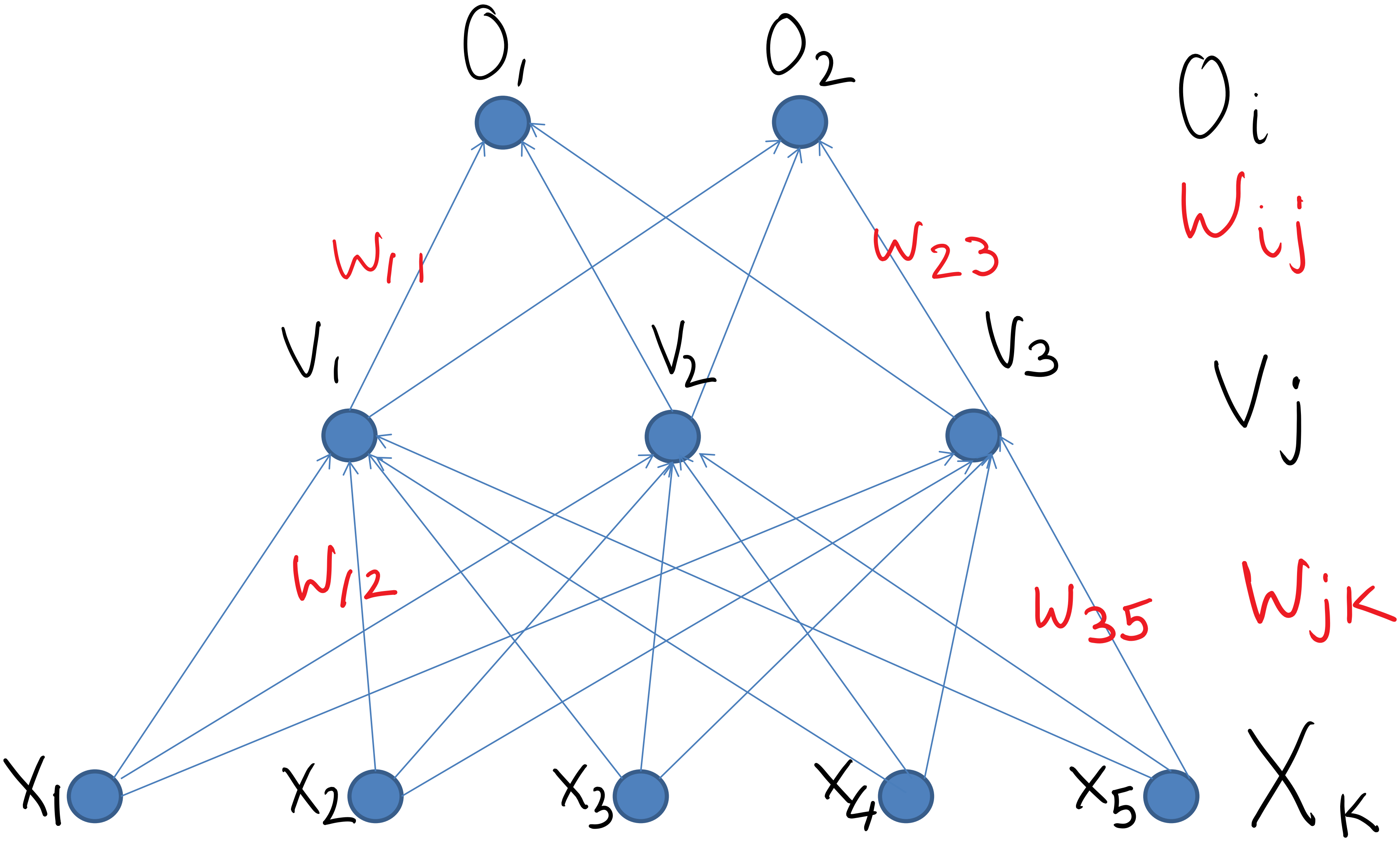


$$V_2 = g\left(\sum_k w_{jk} x_k\right)$$

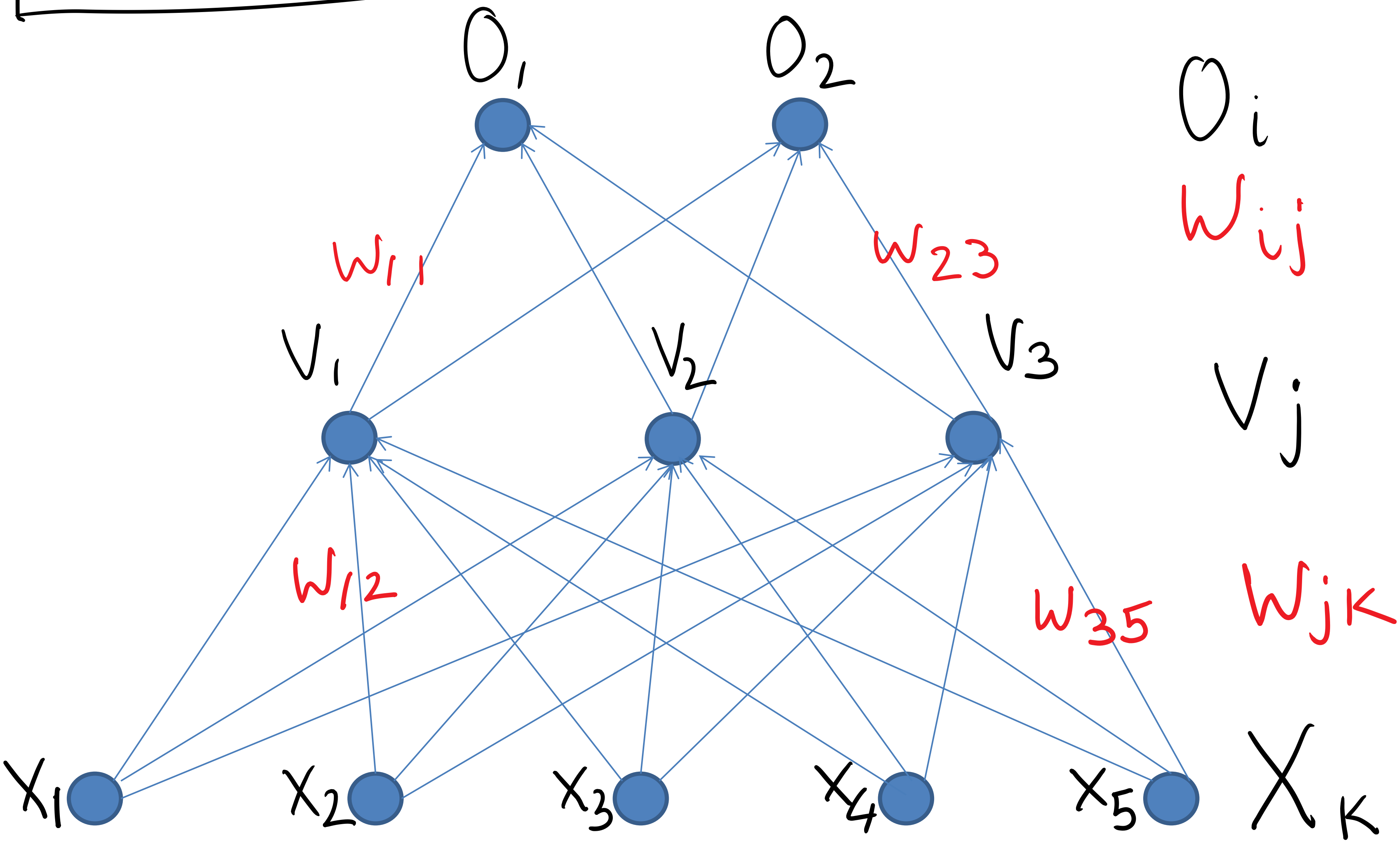
Two layer neural network



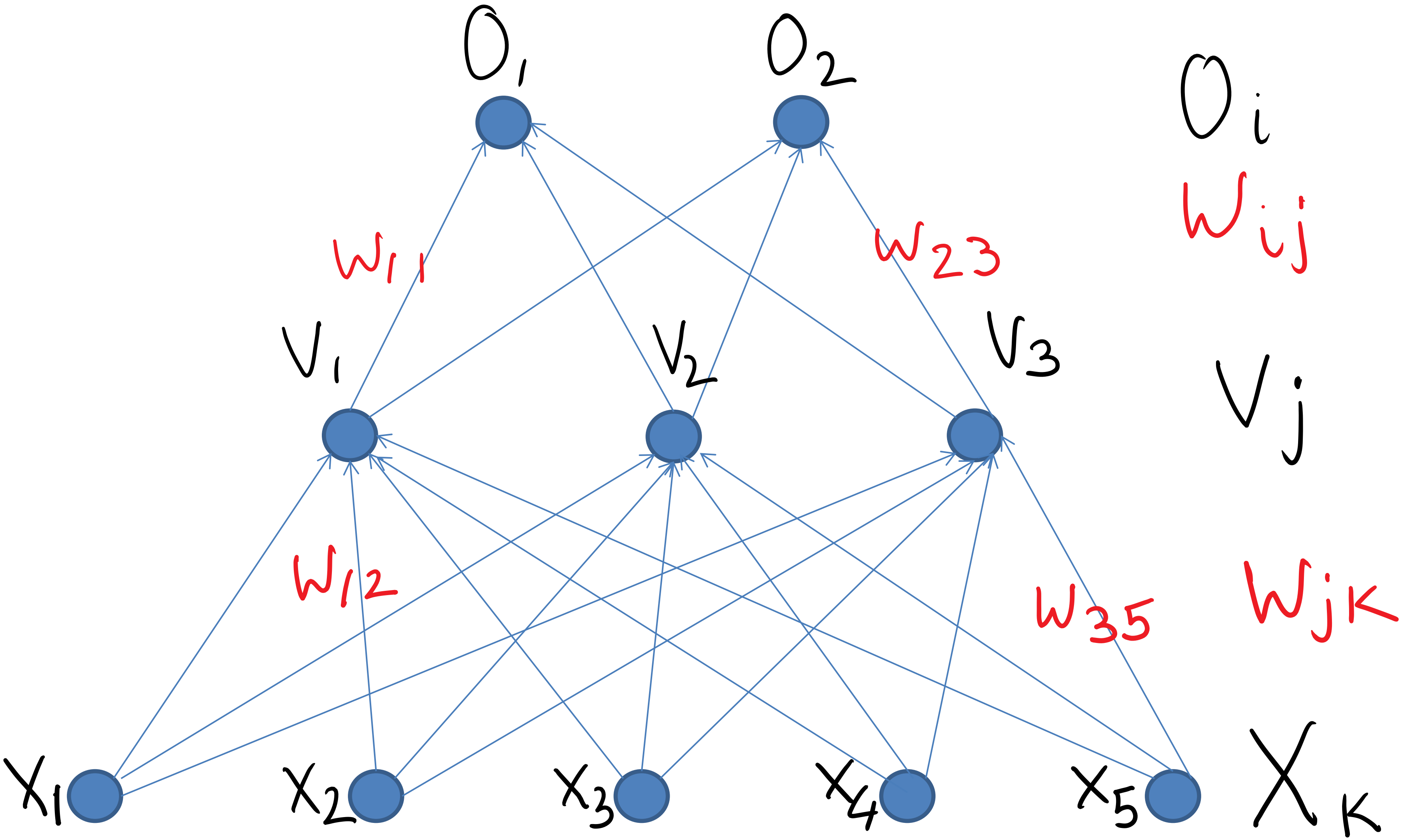
Two layer neural network



$$\boxed{V_j = g\left(\sum_k w_{jk} x_k\right); \quad O_i = g\left(\sum w_{ij} V_j\right)}$$



$$O_i = g\left(\sum_j W_{ij}\, g\left(\sum_k W_{jk}\, x_k\right)\right)$$



Recall from multivariable calculus that the *gradient* of f is the vector

$$\nabla f(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_d} \end{bmatrix}.$$

The gradient is the direction which leads to a maximal increase of f. Similarly, the negative gradient is the direction of *steepest descent*. Gradient descent uses this fact to construct an algorithm: at every step, compute the gradient and follow that direction to minimize f.

Training a neural network

Goal: Find W such that O_i is as close as possible to y_i (desired output)

Approach:
- Define loss function $\mathcal{L}(W)$
- Compute $\nabla_W \mathcal{L}$
- $W_{new} \leftarrow W_{old} - \eta \nabla_W \mathcal{L}$

Training a single layer neural network

- For binary classification, a good choice of loss function is the cross entropy. For regression, use squared error

$$L = -\sum_{\text{input data}} \left(y_i \ln O_i + (1-y_i)\ln(1-O_i) \right)$$

- We model the activation function *g* as a sigmoid

$$g(z) = \frac{1}{1+\exp(-z)}$$

- Finding w reduces to logistic regression!

We can use STOCHASTIC GRADIENT DESCENT.